Graduate Mathematics Courses

Spring 2025

Math 500, Abstract Algebra I

Spring 2025

- **Section:** N1 **CRN:** 38153
- **Time and Place:** MWF at 10:00am in 36 English.
- **Instructor:** Nathan Dunfield
  - **E-mail:** nmd@illinois.edu
  - **Office:** 378 Altgeld
  - **Office Hours:** Wednesdays 1:30-2:30pm and Thursdays 2:00-3:00pm; other times possible by appointment.
- **Web page:** http://dunfield.info/500
- **Detailed schedule with lecture notes, homework, and exams.**

Course Description

This is a graduate course in abstract algebra. The catalog description is:

Isomorphism theorems for groups. Group actions. Composition series. Jordan-Holder theorem. Solvable and nilpotent groups. Field extensions. Algebraic and transcendental extensions. Algebraic closures. Fundamental theorem of Galois theory, and applications. Modules over commutative rings. Structure of finitely generated modules over a principal ideal domain. Applications to finite Abelian groups and matrix canonical forms.

with more details in the official departmental syllabus. This corresponds roughly to Chapters 1-8 and 10-14 of the textbook.

Prerequisites: Undergraduate linear and abstract algebra (basics of groups, rings, fields, vectors spaces, etc.), for example as covered in Math 416 and Math 417.

Required text: Dummit and Foote, Abstract Algebra, 3rd Edition, 944 pages, Wiley 2003. The Grainger Engineering Library has a copy on reserve for in-library use.

Suplemental text: Charles Rezk, Lecture Notes for Math 500, Fall 2022: Part 1 (Groups), Part 2 (Rings and modules), Part 3 (Fields and Galois theory).

Course Policies

Overall grading: Your course grade will be based on homework (40%), two in-class midterm exams (15% each), and a comprehensive final exam (30%). You can view all of your scores in the online gradebook.

Weekly homework: These will typically be due on Friday. They are to be turned in **on paper** at the **start** of the class period. If you are unable to attend due to illness or quarantine, you can email me a PDF single file with a scan of your HW; if using your phone/tablet, please use an app designed for this purpose, such as Abobe Scan (iOS, Android). Late homework will not be accepted; however, your lowest two homework grades will be dropped, so you are effectively allowed two infinitely late assignments. Collaboration on homework is permitted, nay encouraged! However, you must write up your solutions individually and *understand them completely*.

In-class midterms: These two 50 minute exams will be held in our usual classroom on the following Wednesdays: February 26 and April 9 (weeks 6 and 11, respectively).

Final exam: The final exam will be Thursday, May 15 at 1:30pm.

Missed exams: There will typically be no make-up exams. Rather, in the event of a valid illness, accident, or family crisis, you can be excused from an exam so that it does not count toward your overall average. I reserve final judgment as to whether an exam will be excused. *All such requests should be made in advance if possible, but in any event no more than one week after the exam date.*

Cheating: Cheating is taken very seriously as it takes unfair advantage of the other students in the class. Penalties for cheating on exams, in particular, are very high, typically resulting in a 0 on the exam or an F in the class.

Disabilities: Students with disabilities who require reasonable accommodations should see me as soon as possible. In particular, any accommodation on exams must be requested at least a week in advance and will require a letter from DRES.

Math 501, Algebra II, Spring 2025

Igor Mineyev. Math 501, Spring 2025. MWF 3pm.
`https://mineyev.web.illinois.edu/class/25s/501/`

Prerequisite: Math 500 or Math 418 or equivalent or consent of the instructor.

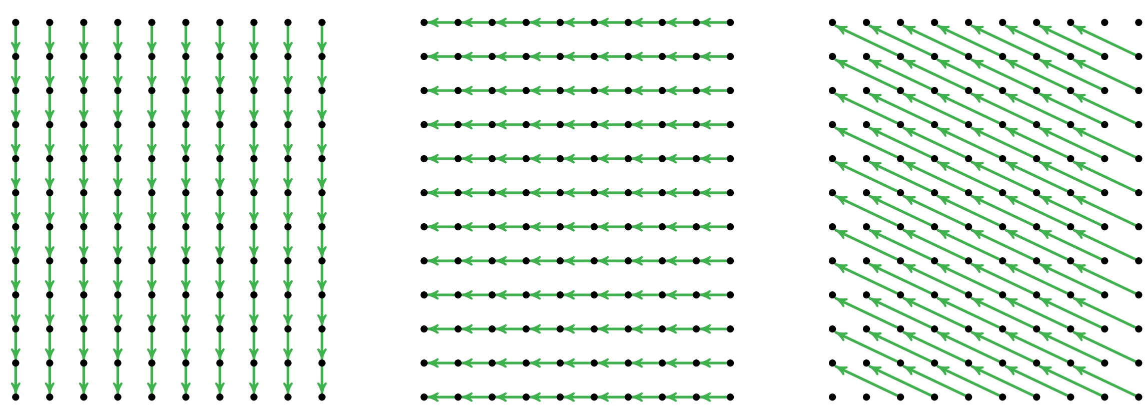
Note that the time of the 501 class will be 3pm MWF (different from the time previously posted in the class schedule).

In the good old days our department used to run a separate course on homological algebra. This is not the case anymore. (Oh, those good old days!) Homological algebra is important: it provides general tools to compute homology and cohomology that are important for multiple other areas of mathematics: algebraic topology, geometry of manifolds, algebraic geometry, group theory, and others.

This fact also gives an opportunity to restructure this 501 course. For this and other reasons, this course will be somewhat experimental. My plan is to generally follow the required syllabus, and also to modify it by including more tools from homological algebra, all the way up to *and including* spectral sequences.

Suggestions and comments on what this course should reasonably cover are also welcome.

There will be some homework and a project in which students will investigate and present topics related to algebra. (For example, possible applications of spectral sequences, or other uses of algebra.)



Math 511. Intro to Algebraic Geometry Lecture Syllabus

The course will prepare students for research in algebraic geometry and related areas including number theory, algebraic combinatorics, logic, and complex geometry.

Topics include:

- Plane Algebraic Curves
    - Smooth curves, local parameter
    - Function field of a curve
    - Rational curves
    - Bezout's Theorem
- Affine algebraic sets and varieties
- Irreducible components
- Regular and rational functions
- Regular and rational maps
- Projective and quasiprojective varieties
    - Projective spaces
    - Hypersurfaces and complete intersections
    - Grassmannians
- Products of varieties
- Normal varieties; normalization
- Dimension
- Singular and nonsingular points; tangent spaces
- Birational maps; blowups
- Divisors
    - Cartier and Weil divisors
    - Linear systems
    - Maps to projective space
- The group law on a plane cubic

Possible texts:
Shafarevich, Basic Algebraic Geometry, Volume 1
Effective Spring 2014. (Approved by GAC, December 2012.)

Math 519 – Differentiable Manifolds II

Spring 2025

This course is the second part of a two-course sequence dedicated to the study of differentiable manifolds. The foundational elements of the theory were covered in Math 518. This course builds upon that foundation and is divided into two parts: the first part focuses on the study of Riemannian manifolds (manifolds with a smoothly varying inner product on the tangent spaces), and the second part addresses more advanced concepts (e.g., vector bundles, principal bundles, connections), with the goal of developing a deeper understanding of smooth manifolds and their geometric structures.

If you did not take Math 518 last semester, please familiarize yourself with the topics covered in that course.

Instructor: Rui Loja Fernandes

Department of Mathematics

Contact Information

- E-mail: ruiloja@illinois.edu
- Office Location: 366 Altgeld Hall (not in use this semester)
- Office Phone: 217-300-2431 (leave message)
- Office Hours: Tue 11:00-11.50AM:

For information about the instructor see the instructor's homepage

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Course Overview

- **Riemannian geometry.** Riemannian metrics, covariant derivatives, parallel transport, geodesics, Hopf-Rinow Theorem. Curvature tensors, first and second variation formulas, Jacobi fields, Myers' and Hadamard's Theorems, Gauss-Bonet Theorem. Hodge-star operator, Laplace operator, harmonica forms, Hodge-de Rham theorem.
- **Fiber bundles.** Vector bundles, principal bundles, connections, parallel transport, curvature, Chern-Weyl theory, Poincaré-Hopf Theorem.
- **Advanced topics:** Either symplectic geometry or Morse theory (time permitting)

Course Location and Time: TBA, MWF 11.00-11.50 am.

Course Goals

The main goals of this course are to:

- Understand the basics of Riemannian metrics and their role in differential geometry.
- Learn the fundamentals of bundle theory and its importance in differential geometry.
- Apply metrics and bundle theory to study global invariants of manifolds.

Recommended Textbooks

- S. Gallot, D. Hulin and J. Lafontaine, Riemannian Geometry third edition. Springer-Verlag, Universitytext 2004.
- R.L. Fernandes, Lectures on Differential Geometry, World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2024.
- J. Jost, Riemannian Geometry and Geometric Analysis 6th Edition, Springer-Verlag, Universitytext 2011.
- S. Kobayashi and K. Nomizu, Foundations of Differential Geometry, vol 1 and 2 New York, Interscience Publishers, 1963.
- C. Taubes, Differential Geometry: Bundles, Connections, Metrics and Curvature Oxford University Press, 2011.

Background Material

For background on algebraic topology see, e.g.:

- William Fulton, Algebraic Topology: A First Course, Springer-Verlag, GTM vol 153, 1995.
- Alan Hatcher, Algebraic Topology, Cambridge University Press, 2002.

Homework Assignments

There will be 5 homework assignments. Homework assignments are due by midnight (Central time) on the dates specified in the weekly overviews unless otherwise noted.

If you need an extension on an assignment because of medical reasons or personal emergencies, you must address the issue with the course instructor. Such accommodations will be made on a case-by-case basis.

Grades

The final grade will be based on the grade of the 5 homework assignments.

Math 522. Lie Groups and Lie Algebras I Instructor Syllabus

(1) A summary of the theory of finite group representations, including the symmetric group. Review of multilinear algebra.

(2) Linear Lie groups - definition and examples. Lie groups as manifolds.

(3) Lie algebras: As the tangent space at the identity to the Lie group, and also the definition as abstract objects. The exponential map. Examples, esp. *sln*, definition of simple Lie algebras.

(4) Representations of Lie groups and Lie algebras: Definition and some examples.

(5) Integration on groups, Peter-Weyl theorem.

MATH 525: Algebraic Topology I

Instructor: James Pascaleff

Spring 2025

Description This is a first course in algebraic topology, the study of topological spaces by means of algebraic invariants. The main topics are the fundamental group, covering spaces, and homology in its singular and simplicial versions.

Textbook Allen Hatcher, *Algebraic Topology,* Cambridge University Press, 2002. The course covers Chapters 0–2 of this book.

Prerequisites Point-set topology: metric spaces, open and closed sets, continuous functions, general topological spaces. Abstract algebra: groups, fields, vector spaces.

Lecture Time and Location MWF 10:00–10:50am in 104 English Building. (To be confirmed.)

Mathematics 527 — Homotopy Theory
Spring 2025
(2-3 MWF)

Instructor: Charles Rezk
Office: 257 CAB
Email: rezk@illinois.edu
Webpage: `https://rezk.web.illinois.edu/`

Course summary:
This is a course on the modern foundations of algebraic topology and homotopy theory, based on the concept of ∞-categories, and motivated by examples.

Course structure:
This will be a lecture style course. For each class period one student will be assigned to take and write up notes, which will be made available to all class members.

Motivation:
In the couple of decades there has been a revolution in the way people talk about homotopy theory, using the language of *higher category theory*. These ideas have begun to infect other areas (including representation theory and algebraic geometry).
The language of higher category theory has proved to be very powerful and flexible, but there are some real drawbacks:

1. It takes a *huge* amount of work to set things up carefully. The basic theory is laboriously developed in two books by Jacob Lurie, which taken together amount almost 2500 pages. And of course, these do not cover everything you need.

2. All of this heavy lifting is largely hidden in modern papers which make use of higher category theory. When you read such a paper, you will see very little of the grind of Lurie's books.

The idea of this course is bridge the gap between the foundations of the subject and its applications to actual problems people care about. Because of the huge amount of material needed to get to applications, the course will have a somewhat different from other graduate courses. For instance, we'll need to avoid giving proofs in many cases (though we will be careful to make precise definitions and statements). I will emphasize examples whenever possible. Sometimes ideas will be motivated by 1-categorical analogues, for which it is much easier to give complete proofs in finite time.

Syllabus:
The topics to be covered are not fixed, but are likely to include the following.

- Model categories (the old foundational approach to homotopy theory).
- Basic notions of quasicategories (the "model" of ∞-categories we will use).
- Straightening/unstraightening and the ∞-category of ∞-categories.
- Accessible and presentable ∞-categories, and the presentable tensor product.
- Monoidal ∞-categories.
- Stable homotopy theory via ∞-categories.

Prerequisites:
The course is meant to be accessible to a reasonably wide audience. Math 525 is the only necessary prerequisite.

MATH 530. ALGEBRAIC NUMBER THEORY – SPRING 2025

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1. Textbooks

- Algebraic Number Theory, J. S. Milne https://www.jmilne.org/math/CourseNotes/ANT.pdf
- Algebraic Number Theory, J. Neukirch

2. Topics Covered

- **Algebraic Background:** Review norm, trace, discriminant, different, integrality, noetherian; Finitely generated torsion-free modules over a PID.
- **Basics:** Number fields, rings of integers being Dedekind domains, integral bases, quadratic and cyclotomic fields.
- **Global theory:** Lattices in $\mathbb{R}^n$, unit theorems, finiteness of class numbers, examples of computing class numbers using Minkowski bound.
- **Local theory:** Completions of $\mathbb{Q}$ (and number fields), Hensel's Lemma with application to nonsolvability of Diophantine equations.
- **Decomposition of Primes:** Kummer's Lemma, inverse different, norm of ideals, discriminant, decomposition group, inertia group, Frobenius automorphism, application to quadratic reciprocity.
- **Introduction of class field theory**

COURSE DESCRIPTION: MATH 532, SPRING 2025

- Time and place: MWF 12pm-12:50pm, 119 English Bldg
- Instructor: Jesse Thorner
- Office hours: By appointment

This is a course on the Chebotarev density theorem, a ubiquitous and powerful generalization of the prime number theorem for arithmetic progressions that lies at the intersection of algebraic and analytic number theory. We will prove effective forms of the Chebotarev density theorem, with and without the assumption of the generalized Riemann hypothesis for Dedekind zeta functions. These results lead to our best understanding of several tough problems, including

(i) Artin's primitive root conjecture;
(ii) for a polynomial f (irreducible over $\mathbb{Z}$), the classification and relative frequencies of the reductions f modulo primes;
(iii) the distribution of Fourier coefficients of holomorphic cuspidal Hecke eigenforms;
(iv) the nonvanishing of central values of modular L-functions in the family of quadratic twists;
(v) distinguishing elliptic curves by their rational points on their reductions modulo p;
(vi) bounds for the ℓ-torsion subgroup of the ideal class group of a number field.

Our main sources will be the notes and research papers that I provide. All sources I will provide will be available at no cost to enrolled students. I will assume that everyone has a strong background in analytic number theory (at the level of Math 531) and the theory of groups and Galois theory (at the level of Math 500) as well as familiarity with number fields (at the level of Math 530). Students who do not have the aforementioned prerequisites can still enroll in the course, but they will be responsible for learning the prerequisite material on their own. Some suggested (ungraded) exercises might be provided, at the instructor's discretion.

Grades will be based on participation and student presentations on topics determined by the instructor. Presentation topics may require facility with both algebraic and analytic number theory.

Course description: Math 540, Spring 2025

TIME AND PLACE: MWF 11-11:50, 137 Henry Administration Bldg
INSTRUCTOR: T. Oikhberg, oikhberg@illinois.edu
Please email me with questions!
OFFICE HOURS: TBD (in person or on zoom), or by appointment, in 33 Computer Applications Building (CAB)

The "target audience" for this course are first year graduate students in Mathematics. The course is mostly concerned with the theory of functions on $\mathbb{R}^n$. Specifically, we plan to cover the following topics:

- Abstract measure theory, Lebesgue measure, and measurable functions.
- Lebesgue theory of integration and convergence theorems.
- Differentiation of functions, functions of bounded variation.
- L_p spaces.
- [Time permitting] The Hilbert space, Fourier series.

Our main textbook will be *G. B. Folland, Real Analysis*, John Wiley & Sons. Other material will be provided as needed. By way of prerequisites, familiarity with Real Analysis (on the level of MATH 447 or an equivalent course) is required.

Exams. There will be one in-class midterm exam in the middle of the semester, and a 3-hour cumulative final exam at the end.

Homeworks will be assigned throughout the semester. The weakest homework score will be dropped.

Grading. Homeworks will be worth 30% of the total grade, the midterm – another 30%, and the final – 40%.

BEST OF LUCK!

Graduate Course Description
Spring 2025
Math 541: Functional Analysis

Instructor: Denka Kutzarova

Time: MWF 2–2:50

Prerequisites: Math 540

Recommended Text: *An Introduction Course in Functional Analysis*, Adam Bowers and Nigel Kalton, Springer 2014 (the book is available online at the UI library).

Course Description.
The recommended book is based on the lecture notes of a course by Nigel Kalton.
Here are some topics that will be covered:

- Some classical sequence and function spaces.
- Hahn-Banach theorem, Haar measure for compact Abelian groups, duals, biduals, adjoint of an operator.
- Baire category theorem, the open mapping and closed graph theorems.
- General topology, topological vector spaces, extreme points, Haar measure on compact groups.
- Compact operators and Fredholm theory.
- Basics of Hilbert spaces.
- Banach Algebras: the spectral radius, commutative algebras.

If time permits, I'll say at the end of the course a few words about nonlinear analysis related to metric geometry. Bilipschitz embeddings into some classical Banach spaces are used in Computer Science and signal processing.

Grades. The course grade will depend on homework assignments, which will be given periodically.

MATH 550 — Dynamical Systems — Spring 2025

Jared Bronski, Mathematics

bronski@illinois.edu

Lecture Time and Location: TBA

Course LMS: `Canvas`

My coordinates: Altgeld Hall 259, 217-244-8218

List of Topics

- Continuous Dynamics – Existence, uniqueness, continuous dependence
- Discrete Dynamics and Maps – Iterated maps, fixed points, stabilty, chaotic behavior
- Linear Differential Equations
  - Jordan normal form
  - Stability of linear systems with constant coefficients
  - Floquet theory and periodic flows
- Nonlinear equations
  - Limit sets and asymptotic behavior
  - Phase portrait and topological methods in 2 and 3 dimensions
  - Periodic orbits and Poincare-Bendixson
- Nonlinear systems near equilibrium – Hartman Grobman theorem
- Structural Stability
  - Smale Horseshoe
  - Hyperbolic systems
- Hamiltonian systems
- Bifurcation Theory

Text(s): We will be using a variety of sources for this course which you **WILL NOT** need to buy:

Arnol'd. *Ordinary Differential Equations.*

Chicone. *Ordinary Differential Equations with Applications.*

Coddington, Levinson. *Theory of Ordinary Differential Equations.*

Devaney. *An Introduction to Chaotic Dynamical Systems.*

Katok, Hasselblatt. *Introduction to the Modern Theory of Dynamical Systems.*

Strogatz. *Nonlinear Dynamics and Chaos.*

Math 553. Partial Differential Equations
Spring 2025

Instructor:

Nikolaos Tzirakis 237 CAB (Computing Applications Building)
Phone: (217) 244-8233
Email: tzirakis@illinois.edu
Mail Box: 250 Altgeld Hall

Course Description:

The course is a basic introduction to the study of partial differential equations. Topics include: first order equations and characteristics, the Cauchy problem, power-series methods, classification, canonical forms, well-posed problems, distributions, linear equations and generalized solutions, Fourier transform methods, the wave equation, Sturm-Liouville problems and separation of variables, Fourier series, the heat equation, integral transforms, Laplace's equation, harmonic functions, potential theory, the Dirichlet and Neumann problems, and Green's functions.

Textbook: Robert McOwen, Partial Differential Equations: Methods and Applications, Prentice-Hall, Inc. (2003).

Lecture Coverage: We will cover chapters 1 through 5. I will use my lecture notes that contain more details on Fourier transform techniques and the theory of distributions.

Grading Policy:

Homework 20%
Midterm Exam 40%
Final Exam 40%

MATH 561: PROBABILITY THEORY I - SPRING 2025

Instructor. Xuan Wu, xuanzw(at)illinois.edu

Time and Place. TR 11:00–12:20 am at 115 David Kinley Hall.

Course Website. Course info is available at the Canvas site.

Contact. By email from your @illinois email, xuanzw(at)illinois.edu, with "Math 561" in the subject.

Office. 357 Altgeld Hall.

Office Hours. TBA

TA. TBA

Textbook. Lecture notes and homework problems will be posted on the course website and they are the main texts of the course. We will mainly follow the textbook

- Probability: Theory and Examples 4th Edition, Cambridge Series in Statistical and Probabilistic Mathematics (2010) by R. Durrett.

Prerequisite.

- Math 540 Real Analysis I - we will review measure theory topics as needed.
- Math 541 is also nice to have but not necessary.

Course Syllabus. This is the first half of the basic graduate course in probability theory. The goal of this course is to understand the basic theory of probability. We will cover the following topics:
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- Random variables and probability spaces;
- Law of Large Number, Central Limit Theorem, Large Deviation Principle;
- Discrete time Martingales;
- Random walks and Markov chains;

‘

Grading. 50% of your grade will be based on weekly homework assignments, 25% will depend on the midterm exam, and 25% will depend on a take home final exam.

Grading Policy. You are encouraged to work together on the homework and discuss them on Canvas, but I ask that you write up your own solutions and turn them in separately. Few problems will be assigned; emphasis will be placed on clear, concise, and coherent writing.

Date: Ver. Nov 12, 2024.

Course Outline — Probabilistic Combinatorics, Math 585, Spring 2025

Professor: Abhishek Methuku

Classes: MWF: 12:00 – 12:50 pm

E-mail: methuku@illinois.edu

Office Hours: After class and by appointments

Web page: https://sites.google.com/view/abhishekmethuku/home.

TOPICS: The Probabilistic Method is a powerful tool for tackling many problems in discrete mathematics. It belongs to those areas of mathematics which have experienced the most impressive growth in the past few decades. This course provides an extensive treatment of the Probabilistic Method, with emphasis on methodology. We will try to illustrate the main ideas by showing applications of probabilistic reasoning to various combinatorial problems. The topics covered in the class will include (but are not limited to) the first 9 chapters of the textbook + many additional topics.

TEXTBOOK: Most of the topics covered in the course appear in the following book:

The Probabilistic Method, by N. Alon and J. H. Spencer, 4th Edition, Wiley

Other topics appear in recent papers.

REQUIREMENTS: There will be about six homework assignments, intended to help the students check their understanding of the material. Each homework will consist of about 6 problems. The first part of the course studies standard methods. There will be an evening exam on it. To make up lost points from the homework, a research paper could be presented in the class. However, in case the class size is above 15, this is not recommended. Class attendance is highly recommended. To excuse a miss, official policy of the university is followed (doctoral note, etc...)

A homework assignment is **30 points**, the test is **120 points**.

GRADING: (80% - : A), (75% - : A-), (70% - : B+), (65% - : B), (60% - : B-), (55% - : C+), (50% - : C), (45% - : C-) etc. Note that the solutions must be typed out, preferably using LaTeX.

Late homework policy: In case the homework is not submitted on time, it could be submitted for the next class, but you lose 10% of the score. However, late homework might be accepted at most twice during the semester from the same student. If there is an official or medical reason, then try to notify me as soon as possible via e-mail.

PREREQUISITES: There are no official prerequisites, but students need the mathematical maturity and background for graduate-level mathematics. For example, basics of linear algebra, probability and graph theory are assumed to be known. Students need to be independent.

RESOURCES: Electronic mail will be the preferred medium for announcements and questions. I will use the e-mail given in the system. Students are requested to use and monitor their illinois.edu e-mails. Some of the communication will also be via Canvas.

Stochastic Processes on Graphs - Spring 2025

Syllabus: Graphs and networks have become an essential part of our daily lives due to the explosion of data and the popularity of social networks. This course aims to introduce and analyze various probabilistic models of random graphs and dynamics on them, emphasizing heuristics on the big picture and background techniques for rigorous proofs. We will consider models of homogeneous and inhomogeneous random graphs, small world, and scale-free networks. Specific topics include:

- Branching Processes and Probabilistic Methods.
- Erdos-Renyi Random Graphs and Phase Transitions.
- Random Graphs with Given Degree Distributions.
- Small-world, Proportional Attachment, and Scale-Free Models.
- Dynamics on Graphs - epidemics, voter model, first-passage percolation, competition models, etc.
- Random optimization, clustering property, and effect of delay.

The emphasis will be on the ideas behind the proofs rather than on slugging through all the details. This course should be a good introduction to the research on complex networks and dynamics on them.

Books:

- Random Graphs and Complex Networks vol I and II, by Remco Van Der Hofstad.
- Random Graph Dynamics, by Rick Durrett.
- Random graphs, by Svante Janson, Tomasz Luczak and Andrzej Rucinski.
- Research articles on mean field first passage percolation, scale-free percolation, long-range first passage percolation, and mean-field random optimization.

Prerequisite: Familiarity with discrete mathematics, basic probability theory, and Markov chain is necessary.

Requirements for students: Students taking the course for credit need to do some of the following:

- Read a recent research paper and present it in class (for the final two weeks of the course),
- scribe several classes,
- solve homework problems given throughout the course,
- do research on some problems I will suggest.

$2 \times 5 \times 31 \times 492467 \times 25794089$ (3)(2 4)(5 7)(1 6 8 10 9)

Graduate Course Description
Spring 2025
Math 595: Anatomy of Integers and Random Permutations

Instructor: Kevin Ford

Time: MWF 3:00-3:50, G36 Foreign Languages Building

Prerequisites: Basic analytic number theory will be very helpful (equivalent of the first half of Math 531; elementary prime number estimates and multiplicative functions). Some knowledge of basic probability will be helpful but not necessary.

Recommended Texts: *Divisors*, by R. R. Hall and G. Tenenbaum, Cambridge Tracts in Math. **90**, paperback edition, 2008. (highly recommended for purchase).
Extensive course notes are posted on my homepage.

Course Description.
Integers factor uniquely into a product of primes, and permutations factor uniquely into a product of cycles. Basic questions one can ask about these structures are

- How many prime factors does a typical integer $n \in [1, x]$ have? What is the distribution of those prime factors?
- How many cycles does a typical permutation of S_n have? How are the lengths of the cycles distributed?

Perhaps surprisingly, there is a close connection between these two problems, both distributions governed by the same probabilistic law. In the first part of the course we will examine carefully, on many scales, the distribution of the prime factors of typical integers and cycle decompositions of typical permutations. We will emphasize the connections between these two structures, stressing probabilistic techniques and ideas. In the second part, we will apply this knowledge to answer questions which about the distribution of divisors of integers, fixed sets of permutations (a subset of $\{1, \ldots, n\}$ which is itself permuted by the permutation) and applications of these bounds. Some examples:

1. How likely is it that an integer has two divisors in a fixed dyadic interval $(y, 2y]$?
2. How likely is it that an integer has two divisors in *some* dyadic interval $(y, 2y]$?
3. How many *distinct entries* are there in and $N \times N$ multiplication table?
4. How likely is it that a random permutation $\pi \in S_n$ contains a fixed set of size equal to k (that is, contains cycles with lengths summing to k)?
5. What is the probability that a random polynomial is irreducible?

Problems 1,2,3 are classical problems of Erdős. Problem 4 generalizes the classical derangement problem ($k = 1$). Problem 5 is an application of the ideas about random permutations, and the answer is "almost all polynomials" under the proper probability measure.

Grades. The course grade will depend on homework assignments, which will be given periodically. There wil be no exams.

Math 595
Floer Theory with Applications

Instructor: Ely Kerman
Lectures: Tuesday-Thursday 11:00 to 12:20

Course Overview. Morse theory is one of the cornerstones of differential topology. It illuminates the relationship between functions, their critical points and the (algebraic) topology of their domains. Floer theory enriches the powerful ideas of Morse theory and extends them to infinite dimensional settings where many of the basic assumptions crucial to Morse theory fail. It was born out of Andreas Floer's remarkable work on the Arnold conjecture concerning the periodic motions of Hamiltonian dynamical systems. Since then Floer theory has been developed extensively and has played a central role in many breakthroughs in symplectic topology, low dimensional topology and adjacent fields. In this course we will study the foundations of Floer theory and discuss some of its remarkable applications.

The first half of the course will focus on the construction of Hamiltonian and Lagrangian Floer theory, as well as the related construction of symplectic cohomology. After a brief review of Morse theory and symplectic geometry we will cover the technical aspects of the compactness, transversality and gluing theory that underpins all versions of Floer theory.

In the second half of the course, we will study applications of Floer theory in a variety of areas. In dynamics, we will study Floer's proof of the Arnold conjecture and Ginzburg's proof of the Conley conjecture concerning the fixed points of symplectic maps. In geometry, we will study Chekanov's proof that Lagrangian submanifolds have nonzero displacement energy. Finally, in topology, we will study Viterbo's isomorphism between the symplectic cohomology of a cotangent bundle and the homology of the free loop space of its base.

Prerequisites. Basic differential topology at the level of Math 518 is recommended.

SPRING 2025, MATH 595, EXPONENTIAL SUMS

INSTRUCTOR: ALEXANDRU ZAHARESCU

Exponential sums play an important role in analytic number theory, and also in a variety of problems arising from other areas. In many cases exponential sums represent the crucial ingredient in the resolution of such problems. Therefore, one can usually improve on the quality of their results if one can master the corresponding exponential sums.

The first part of the course will cover classical material. For this part we will follow selected chapters from Montgomery's book. In the second part of the course we will study some recent publications and preprints. Among other things, we will present new advances in additive number theory, with special emphasis on Goldbach, Waring, and Pollock type problems, and some recent discoveries related to the distribution of zeros of the Riemann zeta function and of more general L-functions. We will also investigate other topics, such as points on curves over finite fields, lacunary sequences, fractional parts of polynomials, and billiards, where exponential sums play a central role.

Prerequisite: MATH 531.

Recommended Textbook:

H. L. Montgomery, *Ten lectures on the interface between analytic number theory and harmonic analysis*, CBMS Regional Conference Series in Mathematics, 84. Providence, RI, 1994.

There will be no exams. Students registered for this course will be expected to give one or more lectures on topics related to the content of the course. In addition, some homework problems will be assigned, and some open problems will be suggested. Each of these open problems, if solved, could lead to a research project and an eventual publication.

Office hours by appointment.

E-mail: *zaharesc@illinois.edu*

MATH 595-SK: STRING THEORY FOR MATHEMATICIANS
Time and location: MWF 10:00-10:50am, 212 Davenport Hall
Second half minicourse
Instructor: Sheldon Katz

The are two primary goals of the course: to formulate the foundations and principles of string theory in the language of graduate-level research mathematics where possible, and to show how string theory has led to new insights into mathematics (and continues to do so). I will explain how physical principles lead to deep insights into mathematics, including amazing conjectures, many of which are now theorems.

I will start with a quick "self-contained review" of physics (i.e. no prior knowledge of physics assumed!): classical mechanics and conservation laws, quantum mechanics and quantum field theory, relativistic invariance.

Then I'll formulate bosonic string theory and see how it leads to a theory of quantum gravity on the physics side, and exotic objects of geometry on the math side (but not so exotic if you're a homotopy theorist). Then I'll describe dimensional reduction, which explicitly demonstates the surprising phenomenon that bosonic string theory on a circle of radius R is indistinguishable from the theory on a circle of radius $1/R$ (in appropriate units adapted to the scale of the string). This is the simplest example of the mirror symmetry phenomenon, where the physical theories associated with two non-isomorphic Riemannian manifolds are indistinguishable.

Then I'll define supersymmetry mathematically and use it to describe superstring theory, which will bring Calabi-Yau manifolds and associated exotic objects into the theory in a natural way. This takes us to the theory of mirror symmetry for Calabi-Yau manifolds and applications.

Additional topics depend on student interest and time constraints.

Prerequisites. No formal prerequisites, but please continue reading. Physics provides an intellectual organizing principle which is very different from what math grad students are used to. In particular, this course does not follow naturally from any graduate course or courses, using concepts from a wide range of areas of pure mathematics. So *mathematical maturity* is essential. Some guidelines: At least some familiarity with manifolds, differential forms, and vector bundles is essential. If you have taken Math 519 (Differentiable Manifolds II) or Math 514 (Complex Algebraic Geometry), you have more than enough background. The course will most draw on ideas from differential geometry, symplectic geometry, algebraic geometry, representation theory, and variational calculus. While no knowledge of physics will be assumed, knowledge of the relevant physics background will be helpful.