Title: The Mathematics Behind Two Puzzles.
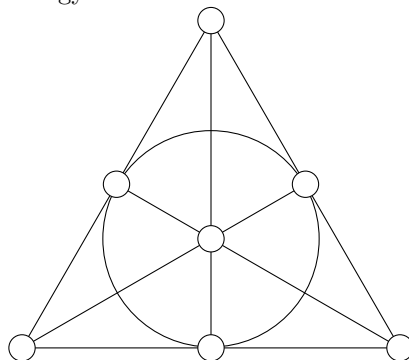
Abstract: I plan to talk about two puzzles with very elegant mathematical solutions. I would encourage you to think about[1] these puzzles, particularly the easier version of the first puzzle, before the talk.

Puzzle 1: You and two friends are teammates on a game show. The three of you are allowed to confer on a strategy beforehand but are not allowed to communicate during the game. The game show assigns a hat color (orange or blue) randomly to each player. Each player is taken to a soundproof room, where they are told the hat colors assigned to their two teammates, but not their own. Each player must then either pass, or attempt to guess their own hat color. Your team wins if at least one player guesses their own hat color correctly and nobody guesses incorrectly. Your team loses if nobody guesses or if there is at least one incorrect guess. Surprisingly there is a strategy that gives you a better than fifty percent chance of winning. Can you find it?

Hard Version: Do the same with seven players.

Puzzle 2: You and 99 friends are being held captive by an evil genie. The genie proposes a game. The captives will be allowed to go one at a time into a room containing 100 boxes, each box containing the name of one captive. Each captive will be allowed to open 50 boxes of their choosing. The genie will release the captives if every captive finds the box containing their own name. **Unfortunately** after opening 50 boxes the captives must leave through a different door and may not communicate what they have seen in the boxes back to the group, and the genie closes all of the boxes after a captive has left the room so it is not possible to convey information to the next captive to enter by changing the state of the room somehow. The genie does, however, allow the captives to confer on a strategy beforehand. If every player opens 50 boxes at random the probability of escape is one in 2^{100} or about one in 8×10^{31}. Can you come up with a strategy to do better?