Math 518 – Differentiable Manifolds 1

Fall 2020

The notion of differentiable manifold makes precise the concept of a space which locally looks like the usual euclidean space $\mathbb{R}^n$. Hence, it generalizes the usual notions of curve (locally looks like $\mathbb{R}^1$) and surface (locally looks like $\mathbb{R}^2$). This course consists of a precise study of this fundamental concept of Mathematics and some of the constructions associated with it: for example, much of the *infinitesimal analysis* (i.e., differential and integral calculus) extends from euclidean space to smooth manifolds. On the other hand, the *global analysis* of smooth manifolds requires new techniques and even the most elementary questions quickly lead to open questions.

Instructor: Rui Loja Fernandes

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Contact Information

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- Office Phone: 217-300-2431 (leave message)
- Office Hours (via zoom): WF 10:00-10.50AM:

For information about the instructor see the instructor's homepage

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Course Overview

- **Foundations of Differentiable Manifolds.** Differentiable manifolds and differentiable maps. Tangent space and differential. Immersions and submersions. Embeddings and Whitney's Theorem. Foliations. Quotients.
- **Lie Theory.** Vector fields and flows. Lie derivatives and Lie brackets. Distributions and Frobenius' Theorem. Lie groups and Lie algebras. The Exponential map. Transformation groups.
- **Differential Forms.** Differential forms and Tensor fields. Differential and Cartan Calculus. Integration on manifolds and Stokes Formula.

Course Location: This course will be fully on-line

Course Lectures Time: Lectures (Zoom sessions) will be held on MWF 9-10 am CST.

Discussion Sessions: Discussion zoom sessions will be held on Mondays at 10 am, following the day's lecture.

NOTE: Course lectures and discussion sessions will be recorded and posted on-line.

Office hours: Office hours via zoom will be held on WF at 10 am, following the day's lecture.

Course Goals

The main goals of this course are:

- Understand what are manifolds and maps between them, and learn how to construct them;
- Understand the symmetry groups of manifolds (Lie groups) and their infinitesimal versions (Lie algebras), and how to work with them;
- Study differential and integral calculus on manifolds, using objects called differential forms.

Recommended Textbooks

You can find the written version of the Lecture Notes here. The recommend textbook is:

- John M. Lee, ***Introduction to Smooth Manifolds***, Springer-Verlag, GTM vol 218, 2nd Ed, 2012. (there is an e-version of this book; see the contents and first chapter here)

Another text that is highly recommended is:

- Michael Spivak, ***A Comprehensive Introduction to Differential Geometry, Vol. 1-2***, (3rd edition) Publish or Perish, 2003.

Background material

For background on multivariable calculus and elementary algebraic topology see, e.g.:

- William Fulton, ***Algebraic Topology: A First Course***, Springer-Verlag, GTM vol 153, 1995.
- James R. Munkres, ***Analysis on Manifolds***, Addison-Wesley (1991), Westview Press (1997).
- James R. Munkres, **Topology**, Prentice Hall, 2000.
- Michael Spivak, ***Calculus on Manifolds: A Modern Approach To Classical Theorems Of Advanced Calculus***, Westview Press 1971.

Homework Assignments

Homework assignments are due by midnight (Central time) on the dates specified in the weekly overviews unless otherwise noted.

If you need an extension on an assignment because of medical reasons or personal emergencies, you must address the issue with the course instructor. Such accommodations will be made on a case-by-case basis.

Homework assignments will be reviewed and graded by the course TA within 1 week. The midterm and final exam will be graded by the course instructor within 5 business days. If your instructor is unable to meet this timeline, students will be notified.

Grades

The final grade will be based on the homework assignments grade (40%), the midterm grade (20%) and the final exam grade (40%). The homework assignments grade will be the average of the 10 best grades of the weekly homework assignments.