
1. a) Use the alternating series test to prove that the series \( S = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \) converges.

b) What is the first \( N \) for which the partial sum \( s_N = \sum_{n=0}^{N} \frac{(-1)^n}{n!} \) approximates \( S \) with error less than 0.0002?

c) Soon we will prove that \( \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{e} \). Use a calculator to compute \( \frac{1}{e} \) and \( S_N \) for the \( N \) above and show that their difference is indeed less than 0.0002.

2. Consider the series

\[
1 - \frac{1}{10^1} + \frac{1}{2} - \frac{1}{10^2} + \frac{1}{3} - \frac{1}{10^3} + \frac{1}{4} - \frac{1}{10^4} + \ldots \quad (\star)
\]

a) Show that the series diverges.

**Hint:** we know that the series

\[
0 + \frac{1}{10^1} + 0 + \frac{1}{10^2} + 0 + \frac{1}{10^3} + 0 + \frac{1}{10^4} + \ldots
\]

converges. If the series (\( \star \)) converged as well, what would happen?

b) Why doesn’t the Alternating Series Test apply to the series (\( \star \))?
3. Use **comparison test** or **limit comparison test** to find out whether the series converge or diverge.

a) \[ \sum_{n=1}^{\infty} \frac{\arctan(n)}{n^3} \]

b) \[ \sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right) \]

c) \[ \sum_{n=3}^{\infty} \frac{1}{(\ln(n))^2} \]

d) \[ \sum_{n=0}^{\infty} \ln(1 + \frac{1}{n^2}) \]

e) \[ \sum_{n=0}^{\infty} \frac{\sqrt{n^2 + 1}}{n^2 + 3} \]

f) \[ \sum_{n=1}^{\infty} \frac{1}{n!} \]