Math 231 Quiz 5, April 1st, 2016

Please show your work for full credit.

1. (7.5 points) Consider the series \( S = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} \). What is the first \( N \) for which the partial sum \( s_N = \sum_{n=1}^{N} \frac{(-1)^n}{n^3} \) approximates \( S \) with error less than 0.01?

\[
b_{n+1} < 0.01 \implies \frac{1}{(n+1)^3} < \frac{1}{100} \implies (n+1)^3 > 100
\]

\[
\begin{array}{c|c|c|c}
N & 2 & 3 & 4 \\
\hline
b_{n+1} & 3^3 = 27 & 4^3 = 64 & 5^3 = 125 \\
125 > 100 & N = 4 \\
\end{array}
\]

2. (7.5 points) Do the series \( \sum_{n=1}^{\infty} \sin\left(\frac{1}{\sqrt{n}}\right) \) converge or diverge? Give complete explanation.

\[
\alpha_n = \sin\left(\frac{1}{\sqrt{n}}\right) \quad b_n = \frac{1}{\sqrt{n}}
\]

\[
\lim_{n \to \infty} \left| \frac{\alpha_n}{b_n} \right| = \lim_{n \to \infty} \frac{\sin\left(\frac{1}{\sqrt{n}}\right)}{\frac{1}{\sqrt{n}}} = \lim_{x \to 0} \sin x = \lim_{x \to 0} \cos x = 1
\]

so \( \sum_{n=1}^{\infty} \sin\left(\frac{1}{\sqrt{n}}\right) \) behaves as \( \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \)

which diverges.