There are nine problems worth a total of 100 points.

Show your work. Circle your answers.

You must not communicate with other students during this test.

No books, notes, calculators, or electronic devices allowed.

Do not turn this page until instructed to.

Violations of academic integrity (in other words, cheating) will be taken extremely seriously, and will be handled under the procedures of Article I, Part 4 of the student code.

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Do not write below this line—for graders only

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1. (18 points) Consider the curve \( y = x^2 + 3 \) between the points \((0, 3)\) and \((1, 4)\).

Set up **but do not evaluate** integrals which represent the following quantities.

a) The length of the curve.

b) The surface area when the curve is rotated about the \( x \)-axis. **Your answer must be an integral with respect to** \( x \).

c) Give another integral which represents the surface area when the curve is rotated about the \( x \)-axis. **Your answer must be an integral with respect to** \( y \).

2. (12 points) If the sequence converges, then give the limit. If it diverges to \( \infty \) then write “\( \infty \)”. Otherwise write “Diverges.” **No explanation required. No partial credit.**

a) \( a_n = \cos(1/n^2) \)

b) \( a_n = \frac{2n^4 + n^3 + 1}{n^4 + 2} \).

c) \( a_n = \frac{\ln n}{\sqrt{n + 1}} \).
3. (8 points) Give mathematical definitions. Each answer should be no more than one line. No partial credit.

a) Define the partial sum \( s_n \) of the series \( \sum_{n=1}^{\infty} a_n \).

b) Define the statement \( \sum_{n=1}^{\infty} a_n = s \).

4. (8 points) Find the sum of the series \( \sum_{n=2}^{\infty} \frac{3^{n-1}}{4 \cdot 5^n} \) or show that it diverges.

5. (8 points) The sum \( s = \sum_{n=1}^{\infty} \frac{1}{n^2} \) is estimated by the 5th partial sum \( s_5 \), with remainder \( R_5 \). Fill in the blanks (your answers should be simplified as much as possible).

\[ \_ \_ \_ \_ \_ \leq R_5 \leq \_ \_ \_ \_ \_ \]
6. (8 points) Determine if the series \( \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{1.1}} \) converges or diverges. Give a complete justification.

7. (15 points) An underwater window has the shape of a triangle whose top edge is 2 meters below the surface. The height of the triangle is 4 meters and the length of the top edge is also 3 meters. Set up but do not evaluate an integral for the hydrostatic force on the window.

Clearly label your coordinates on the axes to the right of the diagram. Use \( \rho \) kg/m\(^3\) for the density of water, and \( g \) m/s\(^2\) for the gravitational constant.
8. (15 points) A lamina with area density \( \lambda \) kg/m\(^2\) occupies the top half of the ellipse \(4x^2 + y^2 = 4\) as shown. You may use the fact that the area of the lamina is \( \pi \) m\(^2\). Find the coordinates \((\bar{x}, \bar{y})\) of the centroid. You may use any available symmetries. Put final answers in the boxes below.

\[ \bar{x} = \underline{\quad} \quad \bar{y} = \underline{\quad} \]

9. (8 points) Consider the integral

\[ \int_{1}^{3} \ln(x)dx \]

Set up the expression which approximates the above integral according to Simpson’s rule with \( n = 4 \) steps. You do not need to simplify.