MATH 285 HOMEWORK 5 SOLUTIONS

SECTION 3.2

22. Imposing the initial conditions \( y(0) = 0 \) and \( y'(0) = 10 \) in the general solution \( y(x) = c_1e^{2x} + c_2e^{-2x} - 3 \) yields the equations \( c_1 + c_2 - 3 = 0 \), \( 2c_1 - 2c_2 = 10 \). The solution is \( c_1 = 4 \), \( c_2 = -1 \). The desired particular solution is \( y(x) = 4e^{2x} - e^{-2x} - 3 \).

23. Imposing the initial conditions \( y(0) = 3 \) and \( y'(0) = 11 \) on the general solution \( y(x) = c_1e^{-x} + c_2e^{3x} - 2 \) yields the equations \( c_1 + c_2 - 2 = 3 \) and \( -c_1 + 3c_2 = 11 \). The solution is \( c_1 = 1 \), \( c_2 = 4 \). The desired particular solution is \( y(x) = e^{-x} + 4e^{3x} - 2 \).

24. The general solution is \( y(x) = c_1e^x \cos x + c_2e^x \sin x + x + 1 \), and its derivative is \( y'(x) = c_1(e^x \cos x - e^x \sin x) + c_2(e^x \sin x + e^x \cos x) + 1 \). Imposing the initial conditions \( y(0) = 4 \) and \( y'(0) = 8 \) on the general solution yields the equations \( c_1 + 1 = 4 \) and \( c_1 + c_2 + 1 = 8 \). The solution is \( c_1 = 3 \), \( c_2 = 4 \). The desired particular solution is \( y(x) = 3e^x \cos x + 4e^x \sin x + x + 1 \).

SECTION 3.3

3. The characteristic equation is \( r^2 + 3r - 10 = 0 \). This factors as \( (r + 5)(r - 2) = 0 \). The roots are \( r = -5, 2 \), which are real and distinct, so the general solution of the differential equation is \( y(x) = c_1e^{-5x} + c_2e^{2x} \).

4. The characteristic equation is \( 2r^2 - 7r + 3 = 0 \). The quadratic formula gives \( r = (7 \pm \sqrt{49 - 24})/4 = (7 \pm 5)/4 = 1/2, 3 \). These are real and distinct, so the general solution of the differential equation is \( y(x) = c_1e^{x/2} + c_2e^{3x} \).

6. The characteristic equation is \( r^2 + 5r + 5 = 0 \). The quadratic formula gives \( r = (-5 \pm \sqrt{25 - 20})/2 = (-5 \pm \sqrt{5})/2 \). The roots are real and distinct, so the general solution of the differential equation is \( y(x) = c_1e^{(-5+\sqrt{5})x/2} + c_2e^{(-5-\sqrt{5})x/2} \).

7. The characteristic equation is \( 4r^2 - 12r + 9 = 0 \). The quadratic formula gives \( r = (12 \pm \sqrt{144 - 4 \cdot 4 \cdot 9})/8 = 12/8 = 3/2 \). Thus there is a single root of multiplicity two. The general solution is therefore \( y(x) = c_1e^{3x/2} + c_2xe^{3x/2} \).

10. The characteristic equation is \( r^4 - 8r^3 + 16r^2 = 0 \). Factoring out \( r^2 \) gives \( r^2(r^2 - 8r + 16) = 0 \), and \( r^2 - 8r + 16 = (r - 4)^2 \). Thus the characteristic equation is \( r^2(r - 4)^2 = 0 \), which has two roots \( r = 0 \) and \( r = 4 \), each with multiplicity two. Therefore the general solution is \( y(x) = c_1 + c_2x + c_3e^{4x} + c_4xe^{4x} \).
15. The characteristic equation is \( r^4 - 8r^2 + 16 = 0 \). Writing this as 
\((r^2)^2 - 8(r^2) + 16 = 0\), we can think of it as a quadratic equation for \( r^2 \). The solution is then \( r^2 = 4 \), and \( r = 2, -2 \), each of which is a repeated root. The full factorization is 
\( r^4 - 8r^2 + 16 = (r^2 - 4)^2 = (r - 2)^2(r + 2)^2 \). The general solution is 
\( y(x) = c_1e^{2x} + c_2xe^{2x} + c_3e^{-2x} + c_4xe^{-2x} \).

39. In order for \( y(x) = (A + Bx + Cx^2)e^{2x} \) to be the general solution, the characteristic equation would need to have a root \( r = 2 \) of multiplicity three. For instance, \((r - 2)^3 = 0\). Expanding this out gives \( r^3 - 6r^2 + 12r - 8 = 0 \). The corresponding differential equation is 
\( y''' - 6y'' + 12y' - 8y = 0 \).