Each problem should be solved on a different page. The maximum grade for this assignment is 100 and the minimum is 0. You can work in pairs/groups, but you should write on your answer sheet with whom you worked.

**Problem 1**

For the following matrices, determine their ranks and null spaces:

(a) \[
\begin{pmatrix}
3 & 0 \\
0 & -1
\end{pmatrix}
\]

(b) \[
\begin{pmatrix}
2 & 0 & 0 \\
0 & -0 & 1 \\
0 & 0 & 0
\end{pmatrix}
\]

(c) \[
\begin{pmatrix}
1 & 0 \\
1 & -1 \\
2 & 1 \\
0 & 1
\end{pmatrix}
\]

(d) \[
\begin{pmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{pmatrix}
\]

(e) \[
\begin{pmatrix}
6 & -2 \\
3 & -1
\end{pmatrix}
\]

(f) \[
\begin{pmatrix}
0 & 0 & -1 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{pmatrix}
\]

**Problem 2**

For the following linear systems, determine (i) the coefficient matrix $A$, (ii) the augmented matrix $(A|b)$, (iii) the associated RREF, (iv) the independent and the dependent variables:

(a) \[
\begin{pmatrix}
3 & 0 \\
0 & -1
\end{pmatrix}
\]

(b) \[
\begin{pmatrix}
2 & 0 & 0 \\
0 & -0 & 1 \\
0 & 0 & 0
\end{pmatrix}
\]

(c) \[
\begin{pmatrix}
1 & 0 \\
1 & -1 \\
2 & 1 \\
0 & 1
\end{pmatrix}
\]

(d) \[
\begin{pmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{pmatrix}
\]

(e) \[
\begin{pmatrix}
6 & -2 \\
3 & -1
\end{pmatrix}
\]

(f) \[
\begin{pmatrix}
0 & 0 & -1 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{pmatrix}
\]
Problem 3

(a) Consider the homogeneous system (in matrix form) $Ax = 0$, where $x \in \mathbb{F}^n$. Prove that the solution set $S$ of the system is a vector subspace of $\mathbb{F}^n$.

(b) Consider the inhomogeneous system $Ax = b$, where $b \neq 0$. Is the solution set $S$ of such system a vector subspace of $\mathbb{F}^n$? Justify your answer.

Problem 4

(a) Let $W$ and $Z$ vector subspaces of a given vector space $V$. Show that the intersection $W \cap Z$ is a vector subspace of $V$.

(b) (FIS Exercise 1.5.15). Let $S_1$ and $S_2$ be subsets of a vector space $V$. Prove that $	ext{span}(S_1 \cap S_2) \subseteq \text{span}(S_1) \cap \text{span}(S_2)$. Give an example in which $\text{span}(S_1 \cap S_2)$ and $\text{span}(S_1) \cap \text{span}(S_2)$ are equal and one in which they are unequal.

Problem 5

Let $M$ be a matrix such that one of its rows is a linear combination of the other rows. Prove that the null space of $M$ is non trivial, i.e. it has more than one element.