Homework Assignment 4

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Due date: 02.17.2016

Each problem should be solved on a different page. The maximum grade for this assignment is 100 and the minimum is 0. You can work in pairs/groups, but you should write on your answer sheet with whom you worked.

Problem 1

Determine whether the following square matrices are either singular or nonsingular.

(a) \[
\begin{pmatrix}
0 & 0 \\
0 & -1
\end{pmatrix}
\]

(b) \[
\begin{pmatrix}
1 & 0 & 1 \\
0 & 1 & 0 \\
0 & 1 & 1
\end{pmatrix}
\]

(c) \[
\begin{pmatrix}
1 & 0 \\
1 & 2
\end{pmatrix}
\]

(d) \[
\begin{pmatrix}
1 & 1 & 0 \\
1 & 0 & 0 \\
1 & 1 & 1
\end{pmatrix}
\]

(e) \[
\begin{pmatrix}
1 & -1 \\
1 & -1
\end{pmatrix}
\]

(f) \[
\begin{pmatrix}
0 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0
\end{pmatrix}
\]

Problem 2

A square matrix \( A \in M_{n \times n}(\mathbb{F}) \) is called upper triangular if all the entries below the main diagonal are zero.

(a) Prove that the set of upper triangular matrices is a vector subspace of \( M_{n \times n}(\mathbb{F}) \)

(b) Prove that an upper triangular matrix \( A \) is nonsingular if and only if all the diagonal entries are different from zero.
Problem 3

Let $A \in M_{n \times n}(\mathbb{F})$ be a square matrix and let $A^t$ denote the transpose of the matrix $A$, namely, $A_{ij}^t := A_{ji}$, $\forall 1 \leq i, j \leq n$.

(a) Prove that $(AB)^t = B^tA^t$.

(b) Prove that $A$ is nonsingular if and only if $A^t$ is nonsingular. *Hint: you may find useful one of the equivalence theorems!*

Problem 4

Determine whether the following sets are either linearly independent or linearly dependent.

(a) $\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \}$

(b) $\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \}$

(c) $\{1 + x^2, 1 - x^2, x^2, 1 + x + x^2\}$

(d) $\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \}$

Problem 5

(a) Find a linearly independent set of 4 elements of $M_{3 \times 3}(\mathbb{R})$.

(b) Prove that if $\{u, v\}$ is a linearly independent set on a vector space over $\mathbb{R}$, then $\{u + v, u - v\}$ is a linearly independent set.