Homework Assignment 5

Instructor: Ivan Contreras
Due date: 03.02.2016

Problem 1
(FIS, Exercise 1.6.31) Let $W_1$ and $W_2$ be finite-dimensional subspaces of a finite dimensional vector space $V$, such that $\dim W_1 = m, \dim W_2 = n$ where $m \geq n$.

(a) Prove that $\dim(W_1 \cap W_2) \leq n$.
(b) Prove that $\dim(W_1 + W_2) \leq m + n$.

Problem 2
Consider the following map:

\[ T : M_{n \times n}(\mathbb{R}) \to M_{n \times n}(\mathbb{R}) \]
\[ A \to A^t \]

(a) Prove that $T$ is a linear transformation.
(b) Find the kernel $N(T)$ and the range $R(T)$.

Problem 3
(a) Prove that there exists a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ such that $T(1, 1) = (1, 2, -1)$ and $T(1, 2) = (1, 0, 0)$.
(b) For the transformation in item (a), determine $N(T), R(T)$ and $T(0, 1)$.
Problem 4

Recall that we checked in class that (counterclockwise) rotations $R_{\theta}$, reflections $T_l$ and projections to the axes $P_x, P_y$ in $\mathbb{R}^2$ are linear transformations.

(a) Prove that the composition $R_{\pi/2} \circ R_{\pi/2}$ is $T_{(0,0)}$, that is, the reflection with respect to the origin.

(b) Give an example of an angle $\theta$ and a line $l$ such that:

$$R_\theta \circ T_l \neq T_l \circ R_\theta.$$

Problem 5

(FIS Exercise 2.1.25)